Stability analysis and model-based control in EXTRAP-T2R with time-delay compensation

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Magnetohydrodynamic instabilities (non-symmetric electric currents) = crucial issue for fusion plasmas & ITER

⇒ Dedicated experiment: Reverse Field Pinch EXTRAP-T2R, with Intelligent-Shell feedback (4 × 32 actuator, 4 × 32 sensor saddle coils)

- MHD control implies
  - aliasing of spatial harmonics induced to actuators and sensors
  - actuation dynamics and control latencies

⇒ New model with experimental constraints and model-based control
EXTRAP T2R fusion plasma experiment

- Machine parameters:
  - major radius $R_0 = 1.24 \text{ m}$
  - plasma minor radius $a = 18 \text{ cm}$
  - shell norm minor radius $r/a = 1.08$
  - shell time constant $\tau_{\text{ver}} = 6 \text{ ms}$
  - plasma current $I_p = 80 \text{ kA}$
  - electron temperature $T_e = 250 \text{ eV}$
  - pulse length $\tau_{\text{pulse}} < 60 \text{ ms}$

Pulse lengths $\tau_{\text{pulse}} \gg \tau_{\text{ver}}$ allow studies of magnetohydrodynamic instabilities that are of the resistive wall mode (RWM) type (growing on the time scale of the shell)
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I. Magnetohydrodynamic unstable modes model

Resistive-wall mode physics in RFP: \textit{from MHD to perturbed ODE}

- Linear stability investigated by periodic spectral decomposition

\[ b(r, t) = \sum_{mn} b_{mn}(r)e^{j(t\omega + m\theta + n\phi)} \]

Fourier eigenmodes $b_{mn}(r)$ with growth-rate $\gamma_{mn} = j\omega_{mn}$,

- Ideal MHD modes:

\[ \tau_{mn}b_{mn}^r - \tau_{mn}\gamma_{mn}b_{mn}^r = b_{mn}^{r,\text{ext}} \]

$b_{mn}^r$: radial component of perturbed field, $b_{mn}^{r,\text{ext}}$: external active coil, $\tau_{mn}$: penetration time

Growth-rates $\tau_w\gamma_{mn}$. Integer-$n$ non-resonant positions (RWMs) are marked (*) for $m = 1$. 

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MIMO plant modeling by geometric coupling of SISO dynamics

- Standard state-space form
  
  - Faraday and Biot-Savart laws + ideal integrator on the sensor coil output voltage (hyp.)

\[
\begin{align*}
\dot{x} &= Ax + Bu + Nv_1 \quad \text{MHD-modes vs. active coil cur. & exog. signal} \\
z &= Mx \quad \text{optional performance vector} \\
y &= Cx + v_2 \quad \text{time-integrated sensor voltages & white noise}
\end{align*}
\]

- with state matrix elements (instantiated for T2R geometry and routing)

\[
\begin{align*}
A_{mn,mnr} &\sim \gamma_{mn}\delta_{mn,mnr} \\
B_{mn,ij} &\sim \tau_{mn}^{-1} \int_{\Omega} e^{-t(m\theta+n\phi)} \left( \hat{r} \cdot \int_{ij} dl_{ij} \times \frac{(r - r_{ij})}{|r - r_{ij}|^3} \right) d\Omega \\
C_{pq,mn} &\sim \int_{\Omega} e^{+t(m\theta+n\phi)} f_{pq}A_{pq} d\Omega
\end{align*}
\]

⇒ physical core of the model.
• Modes coupling and aliasing of spatial frequencies:
  – sensors and actuators: single-mode to multiple-mode model affected by aliasing;
  – the traditional IS-regulator (output convergence) does not counteract the disturbances influence on MHD-modes;
  ⇒ Precise mode-control to address the problem of internal state stability.

• Actuators dynamics, latencies and PID control:
  – Characteristic times (T2R) → actuation
    \[ u_{sys}(t) \approx \frac{1}{\tau_c s + 1} \frac{\kappa}{\tau_d s + 1} u_{DAC}(t - \tau_h) \]
  – PID implementation with time-delay
    \[ u_{DAC}(t) = K_p e(t) + K_i q(t) + \tau_d^{-1} K_d (e(t) - e(t - \tau_d)) \]
  ⇒ Closed-loop dynamics
    \[ \dot{x}(t) = \mathcal{A}_0 \ddot{x}(t) + \mathcal{A}_1(\theta) \ddot{x}(t - \tau_h) + \mathcal{A}_2(\theta) \ddot{x}(t - \tau_h - \tau_d) + \mathcal{E}v_1(t) \]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value/order</th>
<th>Description/comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_{w,N})</td>
<td>13.8 ms</td>
<td>Nominal resistive wall time</td>
</tr>
<tr>
<td>(\tau_w)</td>
<td>(\approx 10) ms</td>
<td>Experimental resistive wall time</td>
</tr>
<tr>
<td>(\tau_{mn})</td>
<td>(\leq \frac{1}{2} \tau_w)</td>
<td>Actual model mode time</td>
</tr>
<tr>
<td>(\tau_{MHD})</td>
<td>(\sim 1) ms</td>
<td>Internal MHD activity/fluctuations</td>
</tr>
<tr>
<td>(\tau_d)</td>
<td>(100) (\mu s)</td>
<td>Digital sampling time, controller cycle</td>
</tr>
<tr>
<td>(\tau_h)</td>
<td>(\sim 100) (\mu s)</td>
<td>Control latency, dead time</td>
</tr>
<tr>
<td>(\tau_{CPU})</td>
<td>&lt; (100) (\mu s)</td>
<td>Algorithm-dependent part of (\tau_h)</td>
</tr>
<tr>
<td>(\tau_a)</td>
<td>(8) (\mu s)</td>
<td>Active amplifier first-order time</td>
</tr>
<tr>
<td>(\tau_c)</td>
<td>(1) ms</td>
<td>Active coil L/R-time</td>
</tr>
<tr>
<td>(\tau_{A&amp;D})</td>
<td>(\sim 1) ms</td>
<td>ADC/DAC settle, ns/(\mu s) respectively</td>
</tr>
</tbody>
</table>
Open-loop error estimation and parameter identification

- Actuators dynamics identification $\rho_{ij} = \{\tau_{c}^{ij}, \tau_{h}^{ij}, k^{ij}\}$ from (PRBS)

$$\rho_{ij}^{*} = \arg \min_{\rho_{ij}} J^{ij}$$

$$J^{2}(\rho_{ij}) = \frac{1}{T} \int_{0}^{T} \left( u_{sys}^{ij}(\tau) - u_{sim}^{ij}(\tau, \rho_{ij}) \right)^{2} d\tau$$

- Error-field estimation and filtering

$$\begin{cases} 
\dot{\tilde{x}} = \left( \begin{array}{cc}
A & N \\
0 & -\tau_{s}^{-1}I
\end{array} \right) \tilde{x} + \left( \begin{array}{c}
B \\
0
\end{array} \right) u + v' \\
y = \left( \begin{array}{cc}
C & 0
\end{array} \right) \tilde{x} + v_2
\end{cases}$$

$$\tilde{x}(t) \doteq (x(t)^{T} x_s(t)^{T})^{T}$$ estimated by Kalman filter, $x_{s}(t)$ is inter alia RWM-instabilities

Experimental data vs. identified model for a particular peripheral channel (top), and filtered estimate of the field-error mode modulus for $(m = 1, n)$: $n = 0, -10, +2$ normalized to $n = 0$ (bottom).
Model summary
II. Stability analysis and delay effects

- Infinite spectrum of the Delay Differential Equation

\[
\det \Delta(s) = \det \left( sI - \mathcal{A}_0 - \sum_{i=1}^{n} \mathcal{A}_i e^{-s\tau_i} \right) = 0
\]

- Mode-control and perfect decoupling: SISO dynamics (fixed gains)

\[
G_{mn}(s) = \frac{1}{\tau_{mn}s - \tau_{mn}\gamma_{mn} \tau_c s + 1} \frac{1}{\tau_a s + 1} e^{-s\tau_h}
\]

→ fictitious but useful for disturbance rejection and resonant-field amplification analysis

- Spectrum dependence on \(\tau_h\): MIMO (rightmost roots) and SISO cases

SISO-set stability in \((\tau_h, \tau_d)\)-space and MIMO-plant stability w.r.t. \(\tau_h\).

⇒ Infinite spectrum model with key peripheral (automation) dynamics.
III. Model-based control and delay compensation

- **Objective:** ensure MHD stability and minimize closed-loop spectral abscissa under PI-TD structure constraints

- **Advantages of TD approach:**
  1. varying computational complexity implies varying $\tau_h$
  2. free $\tau_h$ = fitting parameter to mimic experimental instability onset

**Direct Eigenvalue Optimization**

- direct MIMO approach generally nonconvex and nonsmooth

$\Rightarrow$ hybrid SISO/MIMO method: minimize the maximum spectral abscissas of the SISO set

- *gradient-sampling* method: *robustified* steepest-descent method suitable for nonsmooth optimization
Optimization results

- Two different parameterizations, implicitly assigning the closed-loop performance and control-input norm:
  
  a) varying $k_p$ and searching for the optimal $\tilde{\theta}^* = (k_i^*, k_d^*)$ for a nominal $\tau_h \to$ Maximum value of CL spectrum in $(k_i, k_d)$-space, $(k_i^*, k_d^*)$
  
  b) varying $\tau_h$ and determining the full optimal PID $\tilde{\theta}^* = (k_p^*, k_i^*, k_d^*) \to$ PID optimization parameterised by $\tau_h$

- Robust on the given problems: convergence within 10 – 30 iterations
IV. Experimental results

New T2R experiments: #20743 – #20755 and #20824 – #20838 - \( I_p \approx 85 \text{kA} \), shot length \( \tau_p \approx 50 - 70 \text{ms} \) and reversal and pinch \( (F, \Theta) \approx (-0.27, 1.72) \).

Generic measure of experimental performance

- feedback parameters \( \theta, \nu \in \{y, u\}, \{t_0, t_1\} \): transient or steady-state

\[
J_\nu(\theta) \equiv \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \sigma_\tau \nu^T(\tau, \theta)\nu(\tau, \theta)
\]

- experiment vs. simulations: global shape is conserved = suitable for control

- initial horizontal motion compensated by an external system stronger than IS controller and needs \( \sim 10 \text{ms} \) to enter approximate stationarity
Old PID settings (diamonds) vs. optimized series a) (squares) and b) (circles).
## Performance improvements

<table>
<thead>
<tr>
<th>Shot#</th>
<th>$K_p$</th>
<th>$K_i$</th>
<th>$K_d$</th>
<th>$J_y$</th>
<th>$J_u$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
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<td>20743</td>
<td>150</td>
<td>16000</td>
<td>0.05</td>
<td>0.464</td>
<td>1.66</td>
<td>old setting 1</td>
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<tr>
<td>20744</td>
<td>160</td>
<td>16000</td>
<td>0.04</td>
<td>0.509</td>
<td>1.80</td>
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<tr>
<td>20746</td>
<td>106</td>
<td>37500</td>
<td>0.061</td>
<td>0.259</td>
<td>2.12</td>
<td>series a)</td>
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<tr>
<td>20747</td>
<td>126</td>
<td>47500</td>
<td>0.073</td>
<td>0.304</td>
<td>1.94</td>
<td>a)</td>
</tr>
<tr>
<td>20827</td>
<td>150</td>
<td>16000</td>
<td>0.05</td>
<td>0.501</td>
<td>1.60</td>
<td>old setting 1</td>
</tr>
<tr>
<td>20833</td>
<td>119.6</td>
<td>46800</td>
<td>0.065</td>
<td>0.304</td>
<td>1.77</td>
<td>b)</td>
</tr>
<tr>
<td>20835</td>
<td>106.8</td>
<td>39860</td>
<td>0.058</td>
<td>0.288</td>
<td>1.64</td>
<td>b)</td>
</tr>
</tbody>
</table>

$\Rightarrow$ optimized controllers = 44% reduction of average field energy at the sensors during steady-state period, at the expense of higher input power (+28%).
Conclusions

- New model for MHD instabilities in T2R, explicitly including important geometrical and engineering aspects

- Direct closed-loop PID gain optimization for the corresponding DDE model

  ⇒ experimental intelligent-shell feedback in a RFP fusion research device

- Applicability of the model to real experimental conditions

- Strongly encourage future work, theoretically and experimentally, in both physical modeling and multivariable control for MHD.
References
