SYNTHESIS AND MODELING OF PLASMA VERTICAL SPEED, SHAPE, AND CURRENT PROFILE CONTROL SYSTEMS IN TOKAMAK

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PROBLEM STATEMENT OF PLASMA MAGNETIC CONTROL DURING LIMITER AND DIVERTER PHASES

Vertical cross sections of ITER

Magnetic plasma control system is to:
- track gaps between plasma boundary and 6 reference points moving in space along predetermined trajectories
- track plasma current reference signal
- stabilize plasma vertical speed around zero
EXAMPLES OF ITER PLASMA CONFIGURATIONS

Examples of reference points location

\[ t_1 = 4.61 \text{ s} \quad \text{Initial moment} \]
\[ t_2 = 29.37 \text{ s} \quad \text{Diverter configuration creation} \]
\[ t_3 = 100 \text{ s} \quad \text{Plasma current flattop is reached} \]
LINEAR TIME-VARYING MODEL OF ITER SCENARIO

Linearized model of plasma in tokamak

\[ \frac{dx}{dt} = Ax + Bu + E \frac{dw}{dt}, \quad y = Cx + Fw, \quad y = \left[ \delta g \delta I_{PF} \delta I_p \delta Z_p \delta R_p \right]^T, \quad w = \left[ \delta \beta_p \delta l_i \right]^T \]

**PROPOSAL**

- Create a number of ITER basic linearized models on DINA code
  
  \[ A(t_i), B(t_i), C(t_i), D(t_i), E(t_i), F(t_i), \quad i = 1, \ldots, N \]

- Create of linearized model of the whole ITER scenario by interpolation of basic linear models for time-varying controller design

\[
\begin{align*}
A(t) &= \begin{bmatrix} A(t_{i-1}) & A(t_i) \end{bmatrix} \alpha(t), \\
C(t) &= \begin{bmatrix} C(t_{i-1}) & C(t_i) \end{bmatrix} \beta(t), \\
B(t) &= \begin{bmatrix} B(t_{i-1}) & B(t_i) \end{bmatrix} \alpha(t), \\
D(t) &= \begin{bmatrix} D(t_{i-1}) & D(t_i) \end{bmatrix} \beta(t), \\
E(t) &= \begin{bmatrix} E(t_{i-1}) & E(t_i) \end{bmatrix} \alpha(t), \\
F(t) &= \begin{bmatrix} F(t_{i-1}) & F(t_i) \end{bmatrix} \beta(t), \\
\end{align*}
\]

\[ \alpha(t) = \frac{t_i - t}{t_i - t_{i-1}}, \quad \beta(t) = \frac{t - t_{i-1}}{t_i - t_{i-1}} \]
**FREQUENCY RESPONSES OF TRANSFER FUNCTIONS**

Linear equations used for control system simulation

\[ \frac{dx}{dt} = Ax + [M \ E \ B][w \ v \ u]^T, \quad M = 0, \quad v = \frac{dw}{dt}, \]
\[ y = Cx + [F \ N \ D][w \ v \ u]^T, \quad N = 0, \quad D = 0 \]

Plant model transfer functions

\[ G(s) = C(sI - A)^{-1}[M \ E \ B] + [F \ N \ D] \]

Max and Min singular values of transfer function

\[ \bar{\sigma}(G) = +\sqrt{\lambda_{\text{max}}(G^H G)}, \quad \sigma(G) = +\sqrt{\lambda_{\text{min}}(G^H G)} \]

\[ \bar{\sigma}[G(j\omega)]I_p = 11.5, 12.5, 13.5, 15 \text{ MA}, \quad t_i = 56.21, 63.22, 72.55, 100 \text{ s} \]

\[ \sigma[G(j\omega)] \]
TIME-VARYING CONTROLLER SYNTHESIS

PROPOSAL

• Synthesis of time-varying controller which adapts to current plasma magnetic configurations to achieve maximum stability margin.

• Time-varying controller is the result of interpolation of the set of time-invariant controllers designed at reference points of ITER scenario.

*Time-varying controller is formed from controller transfer functions*

\[
K(s,t) = \alpha(t)K_{i-1}(s) + \beta(t)K_i(s)
\]

\[
\alpha(t) + \beta(t) = 1
\]
PLASMA MAGNETIC CONTROL SYSTEM IN ITER
H_{\infty} SCALAR & MULTIVARIABLE TIME-VARYING CONTROLLERS DESIGNED BY LOOP SHAPING APPROACH

Time-varying DINA-L model, no PF currents variations in feedback
Plant model interpolation of 11.5 MA and 15 MA points at [0, 10] sec
H∞ CONTROLLER DESIGNED BY MIXED SENSITIVITY APPROACH FOR PLASMA CURRENT RAMP-UP PHASE

Time-invariant P and H∞ controllers, PF currents variations in feedback
Plant model interpolation of 11.5 MA and 15 MA points at [5, 10] sec

Gap displacements

CS&PF Currents variations

Control voltages
SINGULAR VALUES FREQUENCY RESPONSES OF OPEN AND CLOSED LOOP SYSTEMS

\[ L_0 = GK \]
\[ S = (I + L_0)^{-1} \]
\[ T = L_0(I + L_0)^{-1} \]
\[ S_0 + T_0 = I \]
\[ \frac{1}{\bar{\sigma}(S_0)} \leq \bar{\sigma}(L_0) + 1 \]
\[ \bar{\sigma}(L_0) \gg 1 \Rightarrow \bar{\sigma}(S_0) \approx \frac{1}{\bar{\sigma}(L_0)} \]
\[ \bar{\sigma}(L_0) \ll 1 \Rightarrow \bar{\sigma}(T_0) \approx \bar{\sigma}(L_0) \]
\[ \bar{\sigma}(\Delta) \leq \frac{1}{\bar{\sigma}(T_0)} \]

Low frequencies \[ \bar{\sigma}(L_0) \gg 1 \Rightarrow \bar{\sigma}(S_0) \ll 1 \] Disturbance rejection
High frequencies \[ \bar{\sigma}(L_0) \ll 1 \Rightarrow \bar{\sigma}(T_0) \ll 1 \] Robust stability
PLASMA SHAPE & CURRENT CONTROL SYSTEMS OPERATION ON DINA CODE

Gap displacements

Control voltages, MPC

Control voltages, $H_\infty$ control

Plasma current variation

CS&PF currents variations, MPC

Plasma vertical speed

CS&PF currents variations, $H_\infty$ control
PLASMA VERTICAL SPEED STABILIZATION AROUND ZERO ON DINA-L MODELS AT SIX SCENARIO POINTS

Scalar system of plasma vertical speed control

Proportional controllers adjusted by trial-and-error methodology

$H_{\infty}$ robust controllers systematically designed on the base of reduced model by loop-shaping approach
IDENTIFICATION OF CLOSED-LOOP SYSTEM OF PLASMA VERTICAL SPEED IN ITER ON DINA CODE

Data for identification

Reference signal and vertical speed, m/s

Time, s

Reference signal
Closed-loop system output

Data for validation

Reference signal and vertical speed, m/s

Time, s

Reference signal
Closed-loop system output

Stable closed-loop transfer function to be identified

\[ \Phi(s) = \frac{K}{(T_1s + 1)(T_2s + 1)} \]

Identification problem statement

\[ V(K, T_1, T_2) = \sum_i \left( \hat{Z}_i - Z_i \right)^2 \rightarrow \min \]

Identification result

\[ K = 3.5654, \ T_1 = 0.11464, \ T_2 = 0.080859 \]
OPEN-LOOP TRANSFER FUNCTION EXTRACTION

Stable closed-loop transfer function obtained by identification

\[ \Phi(s) = \frac{K_P W(s)}{1 + K_P W(s)} \]

Unstable open-loop transfer function obtained from closed-loop system

\[ W(s) = \frac{K_P \Phi(s)}{1 - K_P \Phi(s)} \]

\( K_P \) is gain of proportional controller, \( W(s) \) is plant transfer function

IDENTIFIED TRANSFER FUNCTION OF PLANT MODEL

\[ W(s) = \frac{-17.4838}{(s+30.24)(s-9.152)} = \frac{0.0632}{(-0.1093s + 1)(0.0331s + 1)} \]
VALIDATION OF IDENTIFIED UNSTABLE MODEL IN STABLE CLOSED-LOOP SYSTEM
STABILITY OF SYSTEM WITH P-CONTROLLER

Transfer function of open-loop system

\[ W(s) = \frac{0.0632}{(-0.1093s + 1)(0.0331s + 1)} = \frac{K}{(T_1s + 1)(T_2s + 1)} \]

Closed-loop system with P-controller \( K_p \)

\[ \Phi(s) = \frac{K_pK}{K_pK + (T_1s + 1)(T_2s + 1)} \]

Characteristic equation

\[ T_1T_2s^2 + (T_1 + T_2)s + (1 + K_pK) = 0 \]

Characteristic equation roots

\[ s_{1,2}^* = \frac{1}{2\cdot T_1T_2} \left[-T_1 - T_2 \pm \sqrt{T_1^2 - 2T_1T_2 + T_2^2 - 4T_1T_2K_pK}\right] \]

Condition of closed-loop stability

\[ K_p < -\frac{1}{K} \approx -15.8 \]
STABILITY MARGINS AND ROOT LOCUS
OF CLOSED-LOOP SYSTEM

Nyquist diagram

Bode diagram

Root locus

Phase stability margin = 27.8°
Amplitude stability margin = -3 dB
CONTROLLER DESIGN FOR LINEAR TIME-VARYING PLANT WITH UNCERTAINTY IN PARAMETERS

Linear time-varying plant model

\[ \dot{x}_1 = u, \quad \dot{x}_2 = a(t) x_2 + b(t) x_1 \]

Problem statement: Controller to be synthesized in the negative feedback has to stabilize poles of closed-loop control system.

Plant model non-stationary parameters with uncertainty

Parameters \( a(t) \) and \( b(t) \) are known at seven points \( t_i \) at interval 0 -100 sec.
IDEAL TIME-VARYING CONTROLLER DESIGN

Known plant parameters

\[ a(t) = 0.1 \sin(2\pi/400) \]
\[ b(t) = 1 + \exp(-0.3t) \]

State control law

\[ u = -\left[ k_1(t) \ k_2(t) \right] \left[ \begin{array}{c} x_1 \\ x_2 \end{array} \right]^T \]

Characteristic equation of closed-loop system

\[
\det \left\{ \begin{bmatrix} -k_1(t) & -k_2(t) \\ b(t) & a(t) \end{bmatrix} - sI \right\} = 0
\]

Non-stationary and non-linear controller parameters

\[ k_1(t) = a(t) - 2s_0 \]
\[ k_2(t) = \frac{s_0^2 + (a(t) - 2s_0)a(t)}{b(t)} \]

\( s_0 \) is multiple pole of closed-loop system

Time-varying controller stabilizes dynamics of closed-loop system by stabilization of multiple pole
Closed-loop system with uncertainty

\[ K(t) = \alpha(t)K_i + \beta(t)K_{i+1}, \quad i = 0,1,2...6 \]

Piecewise interpolated controller

Time responses at additive output disturbance \( d \) and various values of \( s_0 \)
INTERPOLATION OF CONTROLLER PARAMETERS

Influence of controller parameters interpolation methodology on accuracy of plant state control

Piecewise interpolation
\[ x_1: 2.8 \% \]
\[ x_2: 3.0 \% \]

Cubic splines
\[ x_1: 1.6 \% \]
\[ x_2: 1.3 \% \]
KINETIC MODEL FOR PLASMA CURRENT PROFILE CONTROL

• Plant controlled model is diffusion equation of magnetic field into plasma (DINA code modification)

• Diffusion equation is solved at fixed plasma current as well as at stationary profiles of plasma temperature and density

• For plasma current profile control five independent current drive sources (actuators) are used

• Plasma current density is measured at five points of tokamak minor radius
TESTING OF PLASMA KINETIC MODEL

Step test actions

- Plant model is a multilink system with distributed parameters
- Testing of plasma model was done by step actions up to achieving stationary values of plasma current density at measured points
**MATRIX OF STATIC COEFFICIENTS**

Static connections of plant “input-output”

Matrix of columns of input signals

Matrix of columns of output signals

initial values

Matrix of columns of steady-state values of output signals

Matrix of static coefficients

\[
Y = KU + Y_0, \quad U, Y, K \in \mathbb{R}^{5 \times 5}
\]

\[
U = \begin{bmatrix}
    u_1 & u_2 & u_3 & u_4 & u_5
\end{bmatrix}
\]

\[
Y_0 = \begin{bmatrix}
    y_0 & y_0 & y_0 & y_0 & y_0
\end{bmatrix}
\]

\[
Y^* = \begin{bmatrix}
    y_1^* & y_2^* & y_3^* & y_4^* & y_5^*
\end{bmatrix}
\]

\[
K = \left( Y^* - Y_0 \right) U^{-1}
\]
Basic control principles

- Decoupling of control channels by means of inverse static matrix $K^{-1}$
- Initial values of output signals are compensated by additive input signals
  \[ u_0 = K^{-1}(y_{ref} - y_0) \]
- Integral units with negative feedback are included in each channel creating astatic diagonal matrix controller
- Multivariable control law
  \[ u = K^{-1} \text{diag}\left[ \frac{k_i}{s} \right] e + u_0 \]
MODEL TRANSFER TO GIVEN PROFILE
AND RETURN TO INITIAL PROFILE

Plasma temperature range at magnetic axis [0.1, 5] keV
MODEL TRANSFER TO GIVEN PROFILE AND FEEDBACK
DISCONNECTION AT RELAXATION PHASE
CONCLUSIONS

- New problem statement of plasma magnetic control as a time-varying parameters plant was formulated.

- Scalar and multivariable time-varying plasma magnetic $H_\infty$ controllers were synthesized and simulated on linearized ITER scenario of DINA code at a portion of plasma current ramp-up phase for plasma vertical speed, shape and current control.

- Plasma vertical speed model was identified on DINA code as a plant of the second order with stable and unstable poles.

- Time-varying controller for plasma vertical speed stabilization around zero was designed and simulated on second order plant model with parameters uncertainty.

- Plasma current profile model and astatic control system with decoupling channels were developed and simulated on DINA code modification.